# WAVELET ANALYSIS OF ONE-DIMENSIONAL TRACTION SIGNALS

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#### Abstract

The article presents issues concerning the analysis of one-dimensional traction signals describing energy conversion processes in driving systems of electric traction vehicles. The method of FFT analysis has been compared to the wavelet method. The most crucial tasks have been discussed of wavelet analyses of signals connected with the problems of drive modelling. Fundamental tasks of coefficient analyses such as approximation, compression and denoising have been taken into account, as well as more complex tasks such as filtering band component and feature extraction. An example of analysis of current signal from tram driving system obtained in the conditions of tractive adhesion failure has been presented. A hypothesis has been demonstrated that coefficients obtained in subsequent stages of wavelet decomposition can constitute the basis for indicating electric current feature vector for the phenomenon of adhesion failure. Application has been proposed of simple characteristics of transform coefficients such as energy, number of zero transitions and power. A feature vector of coefficients has been chosen. A method of feature classification has been proposed in order to implement it in adhesion failure recognition. The results of verification of wavelet analysis application in adhesion failure recognition have been presented. A stance has been taken as to the results of the diagnosis taking into consideration currently used anti-slip systems of traction vehicles.

Keywords: tram driving system with direct current engines, adhesion failure, engine current, recognition

### **1. Introduction**

Applying the notions of signal theory [1] to traction rules, traction signal is understood as process of changes of a selected electromechanical physical value or of a drive state variable in time carrying the information of the course of motion. Mathematical models describing drive operation are usually presented in the form of differential equation sets. In measurement system models, random phenomena of discretization disturbances (sampling and quantization) are also taken into account. As a result, mathematical models of single realizations of traction signals are one-dimensional discrete real-valued functions containing a determined component and noise.

Typical issues of traction signal processing concentrate on extracting information carried by them in order to interpret the process. Effects of obtaining information depend on the manner of signal processing. One of the basic ways of digital processing of stationary signals is FFT transform [2], serving for signal representation in the field of sine and cosine function frequency. In non-stationary signal research, a benchmark method for signal processing is short-term Fourier transform (STFT) [3], making it possible to use time-frequency signal representation. For signal analyses which are not about obtaining information concerning signal spectrum in time, but only about tracing the presence of clearly determined short-term run within the signal, wavelet transforms can be used [4], representing a signal within the field of scale and shift.

The difference between signal analysis by means of Fourier method and the wavelet one, concerns the analysing function. Analysing functions of discrete wavelet transform of a compact carrier are discrete functions determined within a closed range (wavelet and scaling function) [5], [6], whereas Fourier's – are continuous functions determined within infinite range (sine and cosine) [7, 8]. Wavelet transform searches for wavelet shapes within the signal, whereas Fourier's – for sine functions. Analysing functions of wavelet transform are scaled and shifted, whereas

Fourier's – distributed at a uniform rate. Therefore, half of the coefficients of wavelet transform are located within the band of the lowest scale, whereas coefficients of FFT transform cover the whole frequency range at a uniform rate (up to Nyquist frequency).

Figure 1 represents sample diagrams of one-dimensional signal transforms: a) wavelet transform, b) FFT transform. In the example presented, the original signal has 16 elements – wavelet transform three levels of scale (16 elements, 3 of which are non-zero), whereas FFT – nine terms of Fourier sequence (16 elements – all zero). If the purpose of the signal analysis is not its distribution into harmonic ones, but only searching for signal fragments of particular shapes, then the use of wavelet method is more appropriate than FFT method (in the example discussed, it is easier to interpret three coefficients than sixteen).



Fig. 1. Sample diagrams of transforms: a ) wavelet analysis, b) FFT



Fig. 2. General outline of wavelet method applications to feature recognition of one-dimensional traction signals

The article shall present results of wavelet analyses of current applied to recognition of adhesion failure states of trams driven by direct current engines. The signals whose analysis results are presented, refer to adhesion failure during braking period and were measured in Warsaw Trams. The result of analyses concerning adhesion failure during start-up period (with the use of signals measured by the Electrical Engineering Institute) was presented in papers [9, 10].

Figure 2 presents the diagram applied to recognition of the phenomenon with the use of wavelet method of signal analysis. The most significant part of it is extraction of signal features. What is meant by extraction of features is signal analysis directed at reduction of information

included in the signal to the part of information which can be associated with object functioning (in this case, with adhesion failure). Fundamental difficulty in selecting feature vector of adhesion failure phenomenon is that selected features must fulfil requirements concerning such properties as predictability, power, non-imitability or usefulness. In the analysis presented, feature vector discussed in paper [10] has been used.

# 2. Examples of operating current signals from tram driving system for various adhesion conditions

Figure 3 presents sample experimental current runs of the second group of tram engines with direct current drive for running: a) without adhesion failure, b) with adhesion during braking period ( $T_2$ ). Current scale amounts to 100 A/lot. Sampling frequency equals 25 kHz.

Figure 4 shows signal fragment for running with adhesion failure during braking period. In the signal fragment presented, runs of impulse and slow-varying components are visible. Apart from the enumerated components, the signal also comprises a constant component and measurement noise.



*Fig. 3.* Sample experimental runs of current of the second group of engines<sup>1</sup> of a tram with a direct current drive for running: a) without adhesion failure, b) with adhesion failure during braking period  $(T_2)$ 



*Fig. 4.* Selected fragments of current signal from fig. 3b a) current run with adhesion failure during braking period b) run during several impulsing periods

<sup>&</sup>lt;sup>1</sup> Measurement: W. Kozik (Tramwaje Warszawskie - Warsaw Trams)

### 3. FFT and wavelet analyses in application to component separation

Figure 5 presents power spectrum distribution of FFT of current signal presented earlier in Fig. 3b: a) within frequency range up to 2.5 kHz, b) – up to 25 Hz. Fig. 5a shows visible power striae of the signal from impulsing frequency of 300 Hz. In Fig. 5b, we can see a falling character of power distribution of frequency components of the signal without clear local extremes. There are several reasons for broadening of power spectrum of the signal in this particular frequency range. First of all, a slow-varying component can be aperiodic (in non-linear dynamics systems, deterministic component of current run can have frequency changeable in time). Secondly, in the process of driving, several short-term phenomena occur, causing slow changes of current which are similar to each other, and whose spectrum characteristics overlap (features of phenomena imitability). Thirdly, numerical effects of FFT method are present [1], [8] – spectrum leak (because of non-synchronised sampling), aliasing. The drawback of FFT analysis of the signal is lack of time location of frequency features tested.



Fig. 5. FFT power spectrum distribution of experimental signal: a) within frequency range 2.5 kHz, b) – 25 Hz

Wavelet analysis makes it possible to conduct time location testing of changes of values within the signal. The first fundamental signal component is approximation, which in the case of signals coming from electromechanical driving systems, should illustrate the so called set component of processes. Fig. 6 presents a sample result of approximation for the current signal presented earlier in Fig. 3 b: a) fragment of the original signal subjected to transform, b) scaling coefficients distribution, c) approximation as a result of reverse transform of vector of coefficients.

Figure 7 presents a slow-varying signal component: a) wavelet coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained. The component obtained comprises most information concerning periods of occurrence of slow changes of current signal (of dominating frequencies of about 1 Hz).

Figure 8 presents a slow-varying component: a) wavelet coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained.

Figure 9 presents an impulse component (about 300 Hz): a) wavelet coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained.

Figure 10 presents a diagram of an impulse component fragment in time 50 ms, chosen identically to that of Fig. 4. It is visible that the diagram presented can serve for estimation of impulsing parameters: impulsing frequency and impulse filling coefficient.

## 4. Wavelet method of signal feature extraction

The simplest functions of coefficient assessment can be used as measurements of pulse content of slow-varying components, such as energy, power or number of zero-transitions. In research conducted by the present author concerning adhesion failure recognition during start-up period, especially useful turned out to be the dependence between the difference of scaling function energy and the basic wavelet:  $E^D = e_1 - e_2$  [10]. This function has all the required properties: predictability, power, non-imitability and usefulness. In particular, it makes it possible to shorten time basis of acquisition window to the minimum, which causes significant reduction in calculation outlay. Minimal length of acquisition window can be expressed by the following equation:

$$t_{\min}{}^D = \frac{2^{J-D} - 1}{f_{samp}},\tag{1}$$

where:

 $t_{min}^{D}$  - minimal length of acquisition window,

J - maximal depth of analysis of pattern signal,

 $f_{samp}$  - sampling frequency (25 kHz).



Fig. 6. Approximation L = 1 - 5 for current signal presented earlier in fig. 3 b: a) fragment of the original signal subjected to transform, b) scaling function coefficients distribution, c) approximation as a result of reverse transform of vector of coefficients



Fig. 7. Slow-varying component L = 6 - 9: a) wavelet function coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained



Fig. 8. Commutative component L = 10 - 12: a) wavelet coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained



Fig. 9. Impulse component L = 13 - 15: a) wavelet coefficients distribution, b) component as a result of reverse transform of vector of coefficients obtained



Fig. 10. Impulse component L = 13 - 15 as a detail of Fig. 8, to be compared to the run in Fig. 4b

Figure 11 presents sample assessment results. Whereas Fig. 11a, a diagram of a fragment of analysed pattern current signal of dyadic length J = 19 (2<sup>19</sup> elements, presented in a full form in Fig. 3b) with an indicated braking period in which assessment should be conducted of pulse content of a slow-varying component.

Figures 11a – 11d present results of assessment function application on selected levels of slowvarying component D = 6 - 9. The time basis of acquisition window changes from the value of  $t_{min}^{6} = 0.32$  s (on the level of 6) to the value of  $t_{min}^{9} = 0.04$  s (on the level of 9). The number of signal elements taken for analyses from which representation vectors (coefficients) are created fall down to the value of  $2^{13}$  (level 6) to the value of  $2^{10}$  (level 9). It is easy to notice that power and nonimitability of the feature discussed fall down together with the increase of analysis level– the biggest differences of assessment function occur on level D = 6, and the lowest– on level D = 9(on this level, the possibility of erroneous classification is also the highest). For each level of transform there is a possibility of creating a feature pattern. For instance, on level D = 8 it can be the value of assessment function equalling 0.2. Sample classification results conducted on level D= 8 with a feature pattern 0.2 are placed in Fig. 12 (positive classification result was omitted at the moment t = 5 s, because this moment is the moment of braking initiation).

### 5. Summary

Wavelet transforms are dedicated to searching within signals for fragments of determined finite shapes. Therefore, this method can be used as a signal converter in solving recognition problems in which interpretation of Fourier's distribution results is not straightforward (or is too ambiguous). The example of a signal whose features are difficult to assess in the domain of Fourier's transform coefficients is current run of a group of traction engines for the period of adhesion failure of a track vehicle. Wavelet transform deals well with the analysis of this type of run in which the increase of amplitude of a slow-varying component is a one-time phenomenon of aperiodic character.

The results presented show that there is a possibility of detection of adhesion failure of a traction vehicle during braking period on the basis of current pulse classification in the domain of wavelet transform coefficients. Predicted time delay of recognition method with the use of this analysis equals 40 ms. Further research programme should encompass both the implementation of the method presented for DSP processor and drawing up an intervention algorithm in cooperation with existing anti-slip systems of a vehicle.



Fig. 11. Result of application of difference of scaling function energy and basic wavelet:  $E^D = e_1 - e_2$  as measurement of slow-varying component pulse feature: a) original signal, b) filtering level D = 6 with windowing  $t_{min}^{6} = 0.32$  s, c) filtering level D = 7 with windowing  $t_{min}^{7} = 0.16$  s, d) filtering level D = 8 with windowing  $t_{min}^{8} = 0.08$  s, e) filtering level D = 9 with windowing  $t_{min}^{9} = 0.04$  s



Fig. 12. Result of braking adhesion failure recognition on the basis of wavelet current analysis

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